The Numerical Model Behind *Empathy Alpha*

Making a Symmetrical 3D Matrix

Why symmetrical? Why 3D?

The 12-tone matrix is a fascinating approach to music composition. I first encountered this method in college; even if the music didn't speak to me, the organization of number sets and their inversions and retrogrades was oddly logical. And symmetry just seemed integral to it. I started looking at ways to make a truly symmetrical matrix, essentially by constructing the prime—and foundational—row as a symmetrical object from the start. I had no clue if it would be musically useful but the idea of using a numerical structure to determine pitch, transposition, location, and other parameters was appealing. (Separately, I also wanted to find out whether it could be used to govern language and expression...a topic for another day.)

It was only after I had worked out the 9x9 matrix—that is, nine 3x3 number squares—that I learned about the existence of "magic squares". These are little numerical oddities with interesting symmetrical properties.

Later, it occurred to me that I could construct a cube by "reflecting" the matrix for each side using the inversions and retrogrades of each row. It was crude but it was 3-dimensional. I didn't know it at the time but, I had the right idea, just the wrong prime row. I also hadn't considered examining an internal numerical structure for the cube; it felt a bit impenetrable.

Anyway, fast-forward several years: after studying it again, I thought maybe an internal numerical structure wasn't so complicated. Going through the number rows and their permutations, I found that not all the matrices they generated were equal; some had different qualities and one of these proved to be special—this ended up being my "prime matrix".

The 3-dimensional construction took some time to work out but the solution was nine 9x9 matrices arranged along a z-plane. As it turned out, the diagonal axes—a feature of the original 12-tone matrix—was an essential framework on which to build that internal structure. The finished object looked complicated but I found that it was just different arrangements of 3x3x3 "mini-cubes".

Below is the original analysis and methodology (edited).

The Method: Line to Square to Cube

1D: The Line

Making a Symmetrical String of Numbers

Set theory [1] was helpful here. Given that this was intended for music, a standard 12-digit number would seem to make the most sense. But in my earliest experiments I found that a 9-digit row worked better because it achieved symmetry where a 12-digit one couldn't. So the chromatic scale was off the table. What this meant was that the model would be based on a base-9 numeral system.

Starting with integer 5 and working out, each opposing pair summed to 10 (or 1 in modular arithmetical terms). Using *modulo 9* [2] had other benefits too: it made identifying the same properties of the numerical objects across all number sets much easier to spot.

Across all permutations, summing all integers would always produce 45 but, in one case, by partitioning the number into three groups of three digits, like 834, 159, and 672 (FIG. 1.1), each bore a constant of 15 when summed.

[8 3 4] [1 5 9] [6 7 2]

-6 +5 -1 +3 -4 +4 -3 +1 -5 +6

Figure 1.1: A palindrome of addition/substraction.

These number-pair combinations would be really important: 1 and 9, 2 and 8, 3 and 7, and 4 and 6. With the exception of 1 and 9, which combined with the origin (5), the digits in each of the other number-pair sets were shared evenly among them: 8, 3, and 4, and 2, 7, and 6. I also found that one digit from each group could be combined with a digit from each of the other two groups to form three new groups while maintaining the same constant of 15 (TABLE 1.1).

Using mod 9, the numbers in the subsets from Number Set 1 could be produced by adding 4 (or subtracting 5): 1 (+4) 5 (+4) 9, 7 (+4) 2 (+4) 6, and so on. Those from Number Set 2 could be produced by adding 2 (or subtracting 7): 6 (+2) 8 (+2) 1, 9 (+2) 2 (+2) 4, and so on.

Number Set 1	**Number Set 2**
1 5 9	6 8 1
7 2 6	9 2 4
4 8 3	3 5 7

Table 1.1: The two sets of three subsets of numbers with a constant of 15.

Interestingly, three more subsets could be created by grouping the elements that shared the same *position* in each subset (TABLE 1.2): 147, 258, and 369. Unlike the subsets from Number Sets 1 and 2, these "positional" subsets could be produced by adding 3 (or subtracting 6): 1 (+3) 4 (+3) 7, 2 (+3) 5 (+3) 8, and 3 (+3) 6 (+3) 9. These would turn out to be important for understanding the full 3D model.

Positional Number Set
1 4 7
2 5 8
3 6 9

Table 1.2: A set of positional subsets derived from Number Sets 1 and 2.

These sets represented seven of eight possible 3-digit combinations that summed to 15. The exception was a 456 subset which did not appear in either Number Set 1 or 2 or the Positional Number Set, though it would be central to the 3x3 matrix.

I worked through all permutations of subsets from Number Sets 1 and 2 (excluding rotations and reflections) with digit 5 as the origin. Two of these were symmetrical (TABLE 1.3).

Natural form (prime)	Reflected form (retrograde)
1 6 8 3 5 7 2 4 9	9 4 2 7 5 3 8 6 1
2 6 7 1 5 9 3 4 8	8 4 3 9 5 1 7 6 2

Table 1.3: Two symmetrical number sets derived from four number-pair combinations.

This doesn't look like much but one of the number sets (actually, twelve if counting all permutations from rotations and reflections) was unique. By rewriting set 816357492 as a 3x3 matrix in the format [1,1], [2,1], [3,1], [1,2], [2,2], [3,2], etc., it produced an order 3 magic square [3] (evidently a modern representation of the *Lo Shu* square [4]) with a constant of 15 (FIG. 1.2).

Number Set 816357492

8	3	4
1	5	9
6	7	2

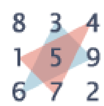

Figure 1.2: Number Set 816357492 configured as a 3x3 magic square.

Looking more closely, every orthogonal—that is, every row and column—in this magic square was identifiable as one of the original subsets that summed to 15: rows 834, 159, and 672, columns 816, 357, and 492, and diagonals 852 and 456. Even the two Positional Number Subsets, 147 and 369, are represented if you trace two overlapping triangles.

Other permutations produced different magic squares—for example, 348726591 and 519384762—but only those with digit 5 at [2,2] produced magic squares where all opposing digits totalled 10 (or 1, *mod 9*). This would become central to building out the 3D matrix.

2D: The Plane

Making a Symmetrical Number Matrix

Collapsing the subsets into 3x3 matrices was a good starting place, but *expanding* them into 9x9 matrices produced structures that were "semi-panmagic" [5] (and some that were not).

Schoenberg's twelve-tone technique [6] was useful here. But where Schoenberg's was rooted in the chromatic scale, here it produced a matrix that was fundamentally identical to the Latin square [7] (and possibly also a reversed Hankel matrix [8]). What different registers do for note transposition in the 12-tone matrix, mod 9 would do for numbers in a 9-digit one: recursion.

Running this on all twelve number sets produced matrices with similar characteristics, all of them associative magic squares [9]. Of all the matrices produced this way, only one of them (not counting its inverted form) had something special (FIG. 2.1): the diagonal perpendicular to the main diagonal (a string of 8s) was made up of the 3-digit subsets from Number Set 1 (159, 267 and 348). Interesting.

Matrix 816357492

8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8

Figure 2.1: An order 9 symmetrical Latin square (Number Set 816357492).

There were a few other surprises: six symmetrical number sets in the 1st, 5th and 9th orthogonals, each row an inversion of a column, and vice versa. Number Set 816357492 and its inversion shared number-pair combinations that summed to 1 (*mod 9*), the number-pair combinations for Number Set 249681735 and its inversion summed to 7, and those for Number Set 573924168 and its inversion summed to 4. The sum of these number-pair combinations aligned exactly with Positional Subset 147. Actually, all six number sets produced symmetrical associative Latin squares with almost identical properties where 1, 4, and 7, 2, 5, and 8, and 3, 6, and 9—the three positional subsets from Number Sets 1 and 2—would be found as constants between all number pairs read from the center of each matrix (from either set of matrices) (FIG. 2.2).

So what does this mean? These can be understood as a permutation group [10] of two sets of three matrices: one natural and one inversion (each form being understood as a rotation of the other about its main diagonal). While any row or column from any matrix could be found in any other matrix (natural or inversion), only the orthogonals in the 1st, 5th and 9th positions are made up of the subsets from Number Set 1 and 2. It means they're uniquely symmetrical.

Matrix Set 1

Matrix 816357492

8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8

Matrix 249681735

2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2

Matrix 573924168

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Matrix Set 2

Matrix 861429375

8	6	1	4	2	9	3	7	5
1	8	3	6	4	2	5	9	7
6	4	8	2	9	7	1	5	3
3	1	5	8	6	4	7	2	9
5	3	7	1	8	6	9	4	2
7	5	9	3	1	8	2	6	4
4	2	6	9	7	5	8	3	1
9	7	2	5	3	1	4	8	6
2	9	4	7	5	3	6	1	8

Matrix 294753618

2	9	4	7	5	3	6	1	8
4	2	6	9	7	5	8	3	1
9	7	2	5	3	1	4	8	6
6	4	8	2	9	7	1	5	3
8	6	1	4	2	9	3	7	5
1	8	3	6	4	2	5	9	7
7	5	9	3	1	8	2	6	4
3	1	5	8	6	4	7	2	9
5	3	7	1	8	6	9	4	2

Matrix 537186942

5	3	7	1	8	6	9	4	2
7	5	9	3	1	8	2	6	4
3	1	5	8	6	4	7	2	9
9	7	2	5	3	1	4	8	6
2	9	4	7	5	3	6	1	8
4	2	6	9	7	5	8	3	1
1	8	3	6	4	2	5	9	7
6	4	8	2	9	7	1	5	3
8	6	1	4	2	9	3	7	5

Figure 2.2: A permutation group of six order 9 symmetrical Latin squares.

And there's something else going on here: if the 1st, 5th and 9th *positions* (a subset from one of the Number Sets) of each orthogonal is understood as a physical marker, we might then expect to find them at the same locations in the other matrices. In fact, we do, and applying the same approach to the positions of all other orthogonals confirms it. And each of these aligns completely with the subsets in Number Set 1: 159, 267, 348.

For example, subset 267 would direct that the same orthogonal should appear at positions 2, 6 and 7, which they do: Row 927468513 at position 6 in Matrix 816357492 was located at position 2 in Matrix 249681735 and position 7 in Matrix 573924168; similarly, column 315864729 at position 3 in Matrix 573924168 was located at position 8 in Matrix 249681735 and position 4 in Matrix 816357492, a direct correlation with subset 348.

Those positions are the building instructions for the 3-dimensional matrix.

You might also have noticed something special about Matrix 573924168 (and its inversion, Matrix 537186942) that the others don't have: *both* diagonals have the same constant of 45. This confirms that Matrix 573924168 (and its inversion) as the most perfect, symmetrical 9-digit structure: An order 9 symmetrical semi-panmagic Latin square (that is also "associative") (FIG. 2.3). That's a mouthful.

Matrix 573924168

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Figure 2.3: Matrix 573924168.

The structure of the 9x9 matrix can be read in several ways (in all its excruciating detail):

Number Subset Composites

As an organization of nine groups of nine numbers (visually, a similar configuration to the popular *Sudoku* [11] number puzzle), three 3x3 Number Subset Composites are arranged symmetrically about the main diagonal (FIG. 2.4). Linked via either digit 2, 5, or 8, each 3x3 composite was a composite of two number subsets, one each from Number Sets 1 and 2. Each 3x3 composite contains three instances of either digit 2, 5, or 8, two instances of two digits from their respective subsets, and one instance each of the two remaining digits in the format AAABBCCDE.

For instance, the 3x3 Number Subset Composite 24679 at position [1,2] has three instances of 2, two instances of 9 and 4 (subset 249 from Number Set 2), and one instance each of 6 and 7 (subset 267 from Number Set 1). Each of the three Number Subset Composites appear in triplicate, each being either a reflection or rotation of another.

Also, the sum of each of the Number Subset Composites always totals 45.

Matrix 573924168

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Figure 2.4: The 2D matrix as an organization of nine 3x3 Number Subset Composites.

Each Number Subset Composite in any of the matrices from Matrix Set 1 or 2 can also be read in the same way as the collapsed 3x3 magic squares: when "zoomed out", you can trace the same overlapping triangles with the same opposing pairs that together total 1 (4 and 6, 9 and 1, etc). Taken as a whole, the organization of numbers in the grid (an example of "tessellation" [12]) can be identified from the Number Subsets, without exception (FIG. 2.5) .

Matrix 573924168

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Figure 2.5: Tessellation of the 3x3 Number Subset Composites.

Combinatorial Number Set Classes

"Combinatoriality"—that is, a group of elements appearing in any combination—is an important device in 12-tone music theory. Similarly, 3-up rows (or columns) can be understood as reflections of themselves about the diagonal (Ie. the retrograde inversion). The blue, black, and red bands, read left-to-right and bottom-to-top, are illustrated in FIG. 2.6 , a recursive pattern that theoretically extends in all directions to infinity.

Natural form

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Retrograde Inversion form

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Figure 2.6: Combinatoriality in the 2D matrix.

Symmetrical Number Regions

By simply dividing the 2D matrix along either diagonal we get the same reflected digits and number pairs as we saw in the 3x3 magic squares (FIG. 2.7).

Primary Axis

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Secondary Axis

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Figure 2.7: The 2D matrix as symmetrical number regions.

3D: The Cube

Making a Symmetrical Number Cube

Using Matrix 573924168 as the first plane, each of the eight successive planes was constructed by organizing the rows and columns such that the position of digit 5 and digit 2 in each plane followed the diagonals between each vertex in the 3-dimensional space. See the red and blue and digits below in FIG. 3.1. The digit 5 and digit 2 vertices are effectively the "scaffolding" on which the number sets are organized.

Plane 1: Matrix 573924168

5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5

Plane 2

3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3

Plane 3

7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7

Plane 4

1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1

Plane 5: Matrix 816357492

8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8

Plane 6

6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6

Plane 7

9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9

Plane 8

4	6	2	8	1	3	9	5	7
2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
3	5	1	7	9	2	8	4	6
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8
8	1	6	3	5	7	4	9	2
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4

Plane 9: Matrix 249681735

2	4	9	6	8	1	7	3	5
9	2	7	4	6	8	5	1	3
4	6	2	8	1	3	9	5	7
7	9	5	2	4	6	3	8	1
5	7	3	9	2	4	1	6	8
3	5	1	7	9	2	8	4	6
6	8	4	1	3	5	2	7	9
1	3	8	5	7	9	6	2	4
8	1	6	3	5	7	4	9	2

Figure 3.1: Nine 2D matrices forming Cube Matrix 573924168.

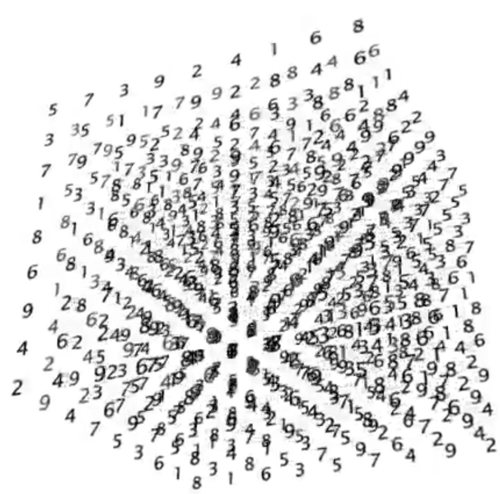

Figure 3.2: Cube Matrix 573924168.

In terms of Cartesian coordinates (*xyz*) the origin (0,0,0) is at digit 8 (or position [5,5] of plane 5). There are many interesting symmetries in the internal structure but the most notable one is that each row or column that intersects the origin can be traced back to the original three number subsets.

The music on the album EMPATHY ALPHA is derived from this model. Each "scene" in the soundtrack is centered on a discrete row, matrix, or cube within the 3D matrix, each deriving its tonality and structure from scalar interpretations of the numerical relationships.

Emlyn Ellis Addison

Written May 18th, 2011, revised January 1st, 2022

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